[a] $\frac{dx}{dy} = \frac{4e^{2y}}{6e^{2y} - 3e^{5x}}$ **FINAL ANSWER**: $3e^{5x} + 4e^{2y} = Ke^{3x}$

$(3e^{5x} - 6e^{2y})\,dx + 4e^{2y}\,dy = 0$

$M = 3e^{5x} - 6e^{2y}, N = 4e^{2y} \Rightarrow M_y = -12e^{2y}, N_x = 0 \Rightarrow \frac{M_y - N_x}{N} = \frac{-12e^{2y}}{4e^{2y}} = -3$ ⬅FUNCTION OF ONLY x

$\mu = e^{\int -3\,dx} = e^{-3x}$

$(3e^{2x} - 6e^{-3x}e^{2y})\,dx + 4e^{-3x}e^{2y}\,dy = 0$ **CHECKPOINT**: $(3e^{2x} - 6e^{-3x}e^{2y})_y = -12e^{-3x}e^{2y} = (4e^{-3x}e^{2y})_x$

$f = \int (3e^{2x} - 6e^{-3x}e^{2y})\,dx = \frac{3}{2}e^{2x} + 2e^{-3x}e^{2y} + C(y)$

$f_y = 4e^{-3x}e^{2y} + C'(y) = 4e^{-3x}e^{2y} \Rightarrow C'(y) = 0$ **CHECKPOINT**: FUNCTION OF ONLY y

$C(y) = 0 \Rightarrow \frac{3}{2}e^{2x} + 2e^{-3x}e^{2y} = K$

[b] $(\cos t)\,dr + (2r^2\cos t - \sin t)r\,dt = 0$ **FINAL ANSWER**: $r^2 = \frac{1}{4\sin t\cos t + C\cos^2 t}$ OR $r^2 = \frac{1}{2\sin 2t + C\cos^2 t}$

$\frac{dr}{dt} + (2r^2 - \tan t)r = 0 \Rightarrow \frac{dr}{dt} - (\tan t)r = -2r^3$ ⬅BERNOULLI

$v = r^{1-3} = r^{-2} \Rightarrow \frac{dv}{dt} = -2r^{-3}\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{1}{2}r^3\frac{dv}{dt}$

$-\frac{1}{2}r^3\frac{dv}{dt} - (\tan t)r = -2r^3 \Rightarrow \frac{dv}{dt} + (2\tan t)r^{-2} = 4 \Rightarrow \frac{dv}{dt} + (2\tan t)v = 4$ ⬅LINEAR

$\mu = e^{\int 2\tan t\,dt} = e^{-2\cos t} = \sec^2 t$

$(\sec^2 t)\frac{dv}{dt} + (2\sec^2 t\tan t)v = 4\sec^2 t$ **CHECKPOINT**: $\frac{d}{dx}\sec^2 t = 2\sec t\sec t\tan t = 2\sec^2 t\tan t$

$(\sec^2 t)v = 4\tan t + C \Rightarrow v = 4\sin t\cos t + C\cos^2 t \Rightarrow r^{-2} = 4\sin t\cos t + C\cos^2 t$

[c] $\frac{dw}{dz} = \frac{w^4 + z^4}{2wz^3}$ **FINAL ANSWER**: $w^2 = z^2 - \frac{z^2}{\ln|z| + C}$

$2wz^3\,dw - (w^4 + z^4)\,dz = 0$ ⬅HOMOGENEOUS since $2(kw)(kz)^3 = k^4(2wz^3)$ and $(kw)^4 + (kz)^4 = k^4(w^4 + z^4)$

$w = vz \Rightarrow 2vz^4(v\,dz + z\,dv) - (v^4z^4 + z^4)\,dz = 0 \Rightarrow 2v(v\,dz + z\,dv) - (v^4 + 1)\,dz = 0$

$\Rightarrow 2vz\,dv - (v^4 - 2v^2 + 1)\,dz = 0 \Rightarrow \frac{dv}{dz} = \frac{v^4 - 2v^2 + 1}{2v} \cdot \frac{1}{z}$ ⬅SEPARABLE

$\Rightarrow \int \frac{2v}{(v^2-1)^2}\,dv = \int \frac{1}{z}\,dz \Rightarrow \frac{1}{1-v^2} = \ln|z| + C \Rightarrow \frac{1}{1-(\frac{w}{z})^2} = \ln|z| + C \Rightarrow \frac{z^2}{z^2 - w^2} = \ln|z| + C$